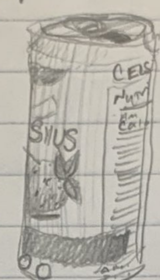
10/28 Lecture Notes

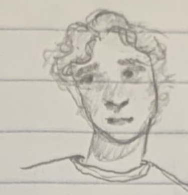
What does an inverse matrix represent?

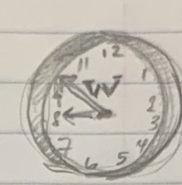
A matrix is invertible if it is both 1-1 & onto
- check for columns spanning all of $\mathbb{R}^n$ (to check if it is
- check if same # of rows & columns
- set = to 0 vector and see what maps to $\begin{bmatrix}0\\0\\\vdots\\0\end{bmatrix}$

identity matrix of corresponding dimensions?

ex: $T(x) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$ is invertible





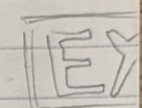


- $T^{-1}(\vec{u}) = \vec{w} \qquad T^{-1}(\vec{v}) = \vec{a}$

  $T(\vec{w}+\vec{a}) = T(\vec{w}) + T(\vec{a}) = \vec{u}+\vec{v}$

Right vs. left inverse?

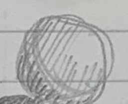
- $T^{-1}(\vec{u}+\vec{v}) = \vec{w}+\vec{a} = T^{-1}(\vec{u}) + T^{-1}(\vec{v})$
- $T^{-1}(c\vec{u}) = cT(\vec{w}) = c\vec{u}$
- $T^{-1}(T(c\vec{w})) = c\vec{w} = T^{-1}(c\vec{u})$ so $cT^{-1}(\vec{u}) = T^{-1}(c\vec{u})$

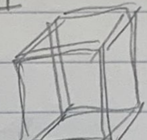


Review matrix multiplication?

Defn: An $n\times n$ matrix A is said to be invertible if there exists an $n \times n$ matrix B such that $AB = I$

Pf. Let $T(x) = A(x)$





Be careful multiplying matrices on the webwork

Finding A^{-1}

Ex) $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$ } $A^{-1} = [\vec{b}_1, \vec{b}_2, \vec{b}_3]$, $A \cdot A^{-1} = I$ } $Ab_1 = e_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\quad Ab_2 = e_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$ $\quad Ab_3 = e_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

let $b_1 = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow$

1st: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right]$

2nd: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 \end{array}\right]$

3rd: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right]$

Row operations won't change — do it all at once!

Look at last class's notes (?)

Instead, write it like this:

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$

put into rref $\Rightarrow$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 4 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{4}{3} & -\frac{2}{3} & \frac{1}{3} \end{array}\right]$

A^{-1}

says that: $b_1 = \begin{bmatrix} -1/3 \\ 2/3 \\ 4/3 \end{bmatrix}$ $\quad b_2 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix}$ $\quad b_3 = \begin{bmatrix} -1/3 \\ 2/3 \\ 1/3 \end{bmatrix}$

so: $\boxed{[A \mid I] \sim [I \mid A^{-1}]}$

if A is invertible

If A is not invertible, you will get a row of all zeroes.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ } shortcut for 2x2 matrix